SPQT2024

Symmetry and Perturbation in Quantum Theory
Santa Margherita di Pula, 2-8 June 2024

Titles and abstracts

(updated on 15/5/2024)

Introductory talks

1. **Horia Cornean** (Aalborg)

   An overview on the magnetic perturbation theory for long range magnetic fields

   A long-range magnetic field (just think about a constant one) comes from a magnetic vector potential which grows at infinity. Such a magnetic perturbation is typically not relatively bounded to the free Laplacian, and proving the stability of gaps in the spectrum of a Bloch-Landau operator is not a trivial task unless your name is Tosio Kato. Also, even though the magnetically perturbed Fermi projections might not be continuous in the norm convergence topology, their local traces are almost always smooth. We will explain (in a very down-to-earth way) how the source of all these apparently contradictory statements can be boiled down to a "divergent" unimodular magnetic phase factor. A very good reference (really an eye-opener, a fantastic work!) is the following paper written in collaboration with Domenico Monaco and Massimo Moscolari: https://arxiv.org/abs/1810.05623. Be there! Don't miss the event!

2. **Roberto Feola** (Roma)

   Long time dynamics for some fluids models

   Wave propagation in dispersive media is described by a large number of partial differential equations, such as the Schrödinger equation (NLS), the wave equation (NLW), the Euler equations of hydrodynamics and the numerous models derived from it. Nonlinear equations are used to describe complex behaviors with coexistence of ordered or chaotic motions depending on the initial data and/or on boundary conditions. The study of these equations poses some fundamental questions that have inspired an entire field of research in recent years: on what time scale do nonlinear solutions remain close to linear ones? Are there new phenomena due to nonlinearity? Is there a "typical" behavior of the solutions? If yes, how long does it persist? During the talk we will discuss some of these problems with particular interest in stability phenomena for solutions of the water wave problem.

3. **Michela Procesi** (Roma)

   Recursive behavior in nonlinear wave models

   Non-linear partial differential equations are effectively used to model waves since they manage to describe their complexity and attempt to give a mathematically rigorous justification of phenomena such as turbulence or the formation of solitary or recursive waves. Keeping water wave models as the main reference and concentrating on recursive solutions, I will give an overview of the main successes of this paradigm as well as the many open problems. I will focus in particular on a natural, but very delicate question, namely whether it is possible to identify a typical behavior for such waves, even only for small and very regular initial data.

4. **Stefan Teufel** (Tuebingen)

   Perturbative construction of non-equilibrium almost stationary states in many-body quantum systems

   I review the construction of non-equilibrium almost stationary states (NEASSs) in interacting many-body quantum systems and then present a new application to the Hall response in insulators at zero temperature and in the thermodynamic limit. Using the NEASS approach, we prove that the Hall conductivity, a linear response coefficient, is given by a many-body version of the double-commutator formula without power-law corrections. The new result is based on joint work Giovanna Marcelli, Tadahiro Miyao, Domenico Monaco, and Marius Wesle.

Regular talks

1. **Fabio Bagarello** (Palermo)

   Some aspects of Quantum Mechanics for non self-adjoint Hamiltonians

   In view of the recent interest in non self-adjoint Hamiltonians in a quantum mechanical context several mathematical (and not only) delicate aspects have been raised. In this talk we consider some of them: biorthogonal families of eigenstates, and bi-coherent states; deformed CCR and other commutation relations; Ladder operators; the role of distributions; an application to the inverted quantum oscillator and to the quantum damped harmonic oscillator; derivations and symmetries for non self-adjoint Hamiltonians.

   arXiv:2001.05219, arXiv:1906.05121, arXiv:2212.01671, arXiv:2204.09968;

2. **Livia Corsi** (Roma)

   Maximal tori in infinite-dimensional Hamiltonian systems

   In the study of close to integrable Hamiltonian PDEs, a fundamental question is to understand the behaviour of "typical" solutions. With this in mind it is natural to study the persistence of almost-periodic solutions and infinite dimensional invariant tori, which are indeed typical in the integrable case. Up to now, almost all results in the literature dealt with very regular solutions for model PDEs with external parameters giving a large modulation. In this talk I shall discuss a new result, constructing Gevrey solutions for models with a very weak parameter modulation. This is a joint work with G.Gentile and M.Procesi.

3. **Francesco Carlo De Vecchi** (Pavia)

   Singular integration by parts formula and quantum field theory

   Characterizing a probability measure through an integration by parts formula is a classical problem in stochastic analysis, finding applications in Euclidean quantum field theory. This problem is closely related to solving the equations of motion for the correlation functions of the quantum field. We focus on this problem in the specific case of quantum field theory with exponential interaction on the plane, where we study a Fokker-Planck-Kolmogorov equation associated with a stochastic quantization equation for such a model. We demonstrate that, under certain conditions on the support of the measure, the solution to this Fokker-Planck-Kolmogorov equation exists and is unique, providing a comprehensive characterization of the exponential measure through an integration by parts formula. This presentation is based on collaborative work with Massimiliano Gubinelli and Mattia Turra.

4. **Maciej Dunajski** (Cambridge)

   Eisenhart metric, and integrability

   There is an old trick, claimed by many, but going back to Eisenhart's work in 1928, for understanding trajectories of a Newtonian particle as geodesic motion in a certain Lorentzian manifold. This sheds light at both classical and quantum integrability. I will report on some recent work with Roger Penrose exploring these issues along side the equivalence principle.

5. **Marco Falconi** (Milano)

   Renormalization group approach to lattice QED

   In this talk, based on an ongoing project with A. Giuliani and V. Mastropietro, I will discuss about the infrared problem in quantum electrodynamics on a lattice. The lattice spacing is kept fixed, and acts as an ultraviolet cut off (preserving gauge invariance). Using Ward identities, we can control the infrared flow of non-gauge-invariant coupling constants – that are generated at intermediate steps in the renormalization group analysis – by means of the gauge-invariant coupling constant e, with the latter vanishing in the infrared (asymptotic freedom). Albeit the scheme is non-perturbative, the results are written as formal power series, and thus make sense only perturbatively.

6. **Frederic Faure** (Grenoble)

   Emergence of quantum dynamics from a classical chaotic dynamics

   The geodesic flow on a closed Riemannian manifold with (strictly) negative curvature is the classical dynamics of a free particle and is very chaotic: it is Anosov, mixing, i.e. any smooth measure of probability on phase space evolving with the dynamics converges weakly towards the uniform (Liouville) measure, called equilibrium. In this talk we will describe the fluctuations around this equilibrium. We will explain that these fluctuations are governed by the wave equation on the manifold, i.e. the quantum dynamics. Techniques for the proofs are micro-local analysis, anisotropic Sobolev spaces and symplectic spinors. Collaboration with Masato Tsujii.

   arxiv:2102.11196

7. **Sören Fournais** (Copenhagen)

   The energy of the dilute Bose gas: A simple approach?

   In this talk, I will present recent progress on the second order correction to the energy of the dilute Bose gas in the thermodynamic limit. By combining the original approach by Fournais and Solovej with recent advances by Haberberger, Hainzl, Nam, Seiringer and Triay on using Neumann bracketing in the localization step, one obtains a much shorter realization of the basic ideas of the original proof. This talk is based on joint work with J.P. Solovej and T. Girardot, L. Junge, L. Morin, M. Olivieri and A. Triay.

8. **Luca Fresta** (Bonn)

   Dynamics of extended Fermi gases at high density

   In my talk, I will discuss the quantum evolution of many-body Fermi gases confined in arbitrarily large domains, focusing on a high-density/semiclassical scaling regime. I will show that, as the density approaches infinity, the many-body evolution of the reduced one-particle density matrix converges to the solution of the Hartree equation, with convergence rate depending on the density only. The result holds for short macroscopic times for non-relativistic particles, but extends to arbitrary times in the case of pseudo-relativistic dispersion. Joint work with Marcello Porta and Benjamin Schlein.

9. **Matteo Gallone** (Trieste)

   Time quasi-periodically driven quantum systems: prethermalization and conservation laws

   Understanding the route to thermalization of a physical system is a fundamental problem in statistical mechanics. When a system is initialized far from thermodynamical equilibrium, many interesting phenomena may arise. Among them, a lot of interest is attained by systems subjected to periodic driving (Floquet systems), which under certain circumstances can undergo a two-stage long dynamics referred to as "prethermalization", showing nontrivial physical features. I will present some prethermalization results for a class of lattice systems with quasi-periodic external driving in time. When the quasi-periodic driving frequency is large enough or the strength of the driving is small enough, we show that the system exhibits a prethermal state for exponentially long times in the perturbative parameter. Focusing on the case when the unperturbed Hamiltonian admits constants of motion, under suitable non-resonance condition we prove the quasi-conservation of a dressed version of them.

10. **Emanuele Haus** (Roma)

    Normal form and dynamics of the Kirchhoff equation

    In this talk I will present some recent results on the Kirchhoff equation of nonlinear elasticity, describing transversal oscillations of strings and plates, with periodic boundary conditions. Computing the first step of quasilinear normal form, we erase from the equation all the cubic terms giving a nonzero contribution to the time evolution of the Sobolev norm of solutions; thus we deduce that, for small initial data of size ε in Sobolev class, the time of existence of the solution is at least of order ε^{-4} (which improves the lower bound ε^{-2} coming from the linear theory). After the second step of normal form, there remain some resonant terms (which cannot be erased) of degree five that give a non-trivial contribution to the time evolution of the Sobolev norm of solutions; this could be interpreted as a sign of non-integrability of the equation. Nonetheless, we show that small initial data satisfying a suitable nonresonance condition produce solutions that exist over a time of order at least ε^{-6}. On the other hand, we use such effective terms of degree five to construct some special solutions exhibiting a chaotic-like behavior. In a more recent work in progress, we also study the normal form of a special Kirchhoff-type equation, which is globally well-posed in time for initial data in Sobolev class. These results were obtained in collaboration with P. Baldi, F. Giuliani, M. Guardia, S. Marrocco.

arXiv:1805.01189, arXiv:2006.01136, arXiv:2007.03543, arXiv:2303.00688

11. **Ian Jauslin** (Rutgers)

    Non-perturbative behavior of interacting Bosons at intermediate densities

    In this talk, I will discuss the behavior of systems of many Bosons interacting via a repulsive, positive-type potential at intermediate densities. The interacting Bose gas has been studied extensively in the low and high density regimes, in which interactions do not play a physically significant role, and the system behaves similarly to (effectively) ideal quantum gases. Instead, I will focus on the intermediate density regime, and report evidence that the system enters a strongly correlated phase where its behavior is markedly different from that of the ideal quantum gas. To do so, I will use the Simplified approach to the Bose gas, which was introduced by Lieb in 1963 and was recently found to provide very accurate predictions for many-Boson systems at all densities.

12. **Boris Konopelchenko** (Lecce)

    (4+4)-dimensional TED equation and integrable Kahler manifolds

    Geometry associated with the integrable (4+4)-dimensional TED equation is discussed. It is shown that a Kahlerian tangent bundle of an affine manifold is a canonical geometric setting for this equation. Particular reductions to the Hessian manifolds and general heavenly equation, self-dual Einstein spaces and Plebanski equations are considered.

13. **Antonella Marchesiello** (Praha)

    Symmetric resonances and related bifurcations

    We consider resonant normal forms corresponding to a wide class of Hamiltonian systems. In particular, we are interested in systems that exhibit symmetries, as e.g. reflectional symmetry with respect to one or more coordinate axes, rotational symmetry or inversion symmetry. We study the bifurcations related to the resonances and the symmetries in their own right, not restricted to natural Hamiltonian systems where H=T+V would consist of kinetic and (positional) potential energy.

14. **Giuseppe Marmo** (Napoli)

    Differential Geometry of Quantum States , with application to Information Geometry

    The talk will be a pedagogical review of work published in the past few years. For finite-level quantum systems, the space of quantum states possesses a very interesting structure: it is a manifold with corners. It has two stratifications, one provided by the action of the unitary group and another one by the complexification of SU(n), namely SL(n,C). This latter action is non-linear and the strata are determined by the rank of the state. By adding an appropriate infinitesimal generator of a semigroup, which changes the rank, we are able to cancel the non-linearity and obtain the generator of a semigroup of completely positive trace-preserving maps. This map, on the dual space, induces a contraction of the associated C^*-algebra and shows how "decoherence" produces a quantum-to-classical transition. The space of states possesses not only a Poisson Bracket but also a compatible Jordan bracket, the pair defines a Lie-Jordan algebra. Two-point distinguishability functions (generalized relative entropies) on the space of states, thought of as probability operators, define alternative Riemannian structures on the space of states satisfying a monotonicity requirement (the distinguishability of probability operators should not increase under coarse graining).

    arXiv:2002.02891, arXiv:1907.00732, arXiv:1903.10465, arXiv:1811.07406, arXiv:1711.09774, arXiv:1711.09769, arXiv:1707.00293, arXiv:1705.05186.

15. **Jessica Massetti** (Roma)

    On linearization of infinite dimensional vector fields

    Given an infinite dimensional vector field $X = \lambda_j x_j(\partial/\partial x^j)$ with $j \in \mathbf{Z}$, $\lambda_j \in \mathbf{C}$, where the frequencies λ_j may satisfy infinitely many resonant relations, we discuss the linearization of holomorphic perturbations $Y = X + P$ and show that Y can be put in some appropriate normal form such that, when restricted to the resonant manifold, the flow is linear with the same characteristic exponents λ_j. This is a joint work with M. Procesi and L. Stolovitch.

16. **Domenico Monaco** (Roma)

    From charge to spin: Analogies and differences in quantum transport coefficients

    We review some recent results from the mathematical theory of transport of charge and spin in gapped crystalline quantum systems. The emphasis will be on transport coefficients, such as conductivities. Those are computed as appropriate expectations of current operators in a non-equilibrium almost-stationary state (NEASS), which arises from the perturbation of an equilibrium state by an external electric field. While for charge transport the usual double-commutator Kubo formula is recovered (also beyond linear response), formulas for appropriately defined spin conductivities are still explicit but more involved. The talk will be based on a survey article, joint with G. Marcelli.

arXiv:2203.08044

17. **Franco Oliveri** (Messina)

    Cooperation and competition in a network of agents modeled by fermionic operators

    An operatorial model of a system made by N agents interacting each other with mechanisms that can be thought of as cooperative or competitive is presented. We associate to each agent an annihilation, creation and number fermionic operator, and interpret the mean values of the number operators over an initial condition as measures of the agents' wealth status. The dynamics of the system is assumed to be ruled by a Hermitian Hamiltonian operator H, and the classical Heisenberg view is used. The dynamical outcome is then enriched by using the recently introduced variant of (H, ρ)-induced dynamics, where ρ denotes a rule that periodically modifies some of the parameters involved in H. The agents are partitioned in three subgroups, one interacting each other only with a competitive mechanism, one interacting each other only with a cooperative mechanism, and one opportunist subgroup able to compete and cooperate. Some numerical simulations show that the (H, ρ)-induced dynamics approach makes, in all the cases, the cooperative subgroup definitely is more efficient in improving its wealth status than the other subgroups. New results are obtained adding a rule that changes the strength of the interactions among the agents.

    doi.org/10.1007/s10773-023-05492-9; arXiv:1803.11234

18. **Peter Pickl** (Tuebingen)

    Beyond Bogoliubov Dynamics

    Consider a system of N interacting bosons which are initially in a product state. It is well established that, in the mean-field scaling regime, product structure remains approximately true under time evolution and that each particle is described by a non-linear wave equation, the Hartree equation. The next order corrections to this mean-field regime are, as proposed by Bogoliubov, given by pair-excitations which evolve naturally according to the Bogoliubov equation. Together with Lea Bossmann, Sören Petrat and Avy Soffer we could construct corrections to the Bogoliubov dynamics that approximate the true N-body dynamics in norm to arbitrary precision. The N-independent corrections are given in terms of the solutions of the Bogoliubov and Hartree equations. In this way, the complex problem of computing all n-point correlation functions for an interacting N-body system is essentially reduced to the problem of solving the Hartree equation and the PDEs for the Bogoliubov two-point correlation functions.

19. **Antonio Ponno** (Padova)

A Klein-Gordon-Wave model for scalar ultralight dark matter

One of the current ideas on the nature of dark matter considers the existence of neutral, spinless particles having mass of the order of 10^{-24}eV (about 10^{-56}g), a hypothesis introduced in the nineties and bringing the name of ultralight scalar dark matter, whose dynamics is usually described by Schroedinger-Poisson-like systems. Sin was perhaps the first to show that the spherically symmetric, stationary solutions of such models might explain the main features of the galaxy rotation curves (considered the main evidence for the existence of dark matter since the pioneer works of Rubin). In such a perspective, galaxies would be held together by huge Bose-Einstein-Condensates of dark matter, being thus objects of a deep quantum nature. Within such a framework, we consider a Klein-Gordon-Wave system describing the field of the dark particle coupled with that of a the "mediator" (a spin zero, toy graviton). Such a Lorentz invariant, Hamiltonian model, in absence of baryonic matter, is ruled by one effective parameter only. We focus first our attention on the study of the stationary radial states of the model, both on the analytical and on the numerical side, including comparisons with the rotation curves. We then show that, if the ruling parameter of the theory is large, i.e. if the particle mass is smaller than a certain threshold-range, a perturbative treatment of the problem is possible, and the Hamiltonian normal form of our model becomes the Schroedinger-Wave system usually considered in the literature. We interpret this as a possible explanation of the mechanisms leading to cold (non relativistic) dark matter. This is a joint work with G. Marangon and L. Zanelli.

20. **Andrea Sacchetti** (Modena)

Rayleigh-Schroedinger perturbation theory for nonlinear Schroedinger equations

We consider a nonlinear perturbation of a time-independent linear Schroedinger equation with isolated simple eigenvalues. It is proved that the associated Rayleigh-Schroedinger power series is convergent when the parameter representing the intensity of the nonlinear term is less in absolute value than a threshold value, and it gives a stationary solution to the nonlinear Schroedinger equation. This result is then applied to a simple model.

https://iopscience.iop.org/article/10.1088/1361-6544/acfdec

21. **Libor Snobl** (Praha)

Integrable systems of the ellipsoidal, paraboloidal and conical type with magnetic field

We construct integrable Hamiltonian systems with magnetic fields of the ellipsoidal, paraboloidal and conical type, i.e. systems that generalize natural Hamiltonians separating in the respective coordinate systems to include nonvanishing magnetic field. In the ellipsoidal and paraboloidal case each this classification results in three one-parameter families of systems, each involving one arbitrary function of a single variable and a parameter specifying the strength of the magnetic field of the given fully determined form. In the conical case the results are more involved, there are two one-parameter families like in the other cases and one class which is less restrictive and so far resists full classification.

22. **Kasper Sörensen** (Roma)

    Bulk-edge correspondence for unbounded Dirac-Landau operators

    We consider two-dimensional unbounded magnetic Dirac operators, either defined on the whole plane, or with infinite mass boundary conditions on a half-plane. We show the so-called Streda formula: if the bulk operator has an isolated compact spectral island, then the integrated density of states of the corresponding bulk spectral projection varies linearly with the magnetic field as long as the gaps between the spectral island and the rest of the spectrum are not closed, and the slope of this variation is given by the Chern character of the projection. The same bulk Chern character is then related to the number of edge states which appear in the gaps of the bulk operator. This is joint work with Horia Cornean and Massimo Moscolari.

23. **Shulamit Terracina** (Milano)

    Reducibility and stability for the Klein Gordon equation with a perturbation of maximal order

    We prove the global in time existence and the stability of the solutions of a class of quasi-periodically forced linear wave equations on the circle of the form $u_{tt} - u_{xx} + mu + Q(\omega t)u = 0$, where Q is an unbounded pseudo-differential operator of order 2, parity preserving and reversible, and the forcing frequency ω belongs to a Borel set of asymptotically full measure. This result is obtained by reducing the Klein-Gordon equation to constant coefficients, applying first a pseudo-differential normal form reduction and then a KAM diagonalization scheme. A central point is that the equation is equivalent to a first order pseudo-differential system which, at the highest order, is the sum of two backward/forward transport equations, with non-constant coefficients, respectively on the subspaces of functions supported on positive/negative Fourier modes. The key idea is to straighten such operator through a novel quantitative Egorov analysis. A main point of interest, in view of applications to a nonlinear setting, is that the change of variables that reduce the equation satisfies tame bounds. This is a joint work with M. Berti, R. Feola and M. Procesi.

24. **F. Verhulst** (Utrecht)

Non-integrability of 3 DOF Hamiltonian systems

Reviewing a number of methods to prove non-integrability of Hamiltonian systems, we focus on 3 degrees of freedom systems listing the known results for the prominent resonances. Associated with the Hamiltonian systems are the averaged-normal forms that provide us with geometric insight, approximations of orbits and measures of chaos. Symmetries do change the qualitative and quantitative picture; we illustrate this for the $1:2:1$ resonance with strong indications of the presence of Shilnikov bifurcations.